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Microcanonical phases of string theory on AdS$_m\times$S^n

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Abstract: Banks, Douglas, Horowitz and Martinec [1] recently argued that in the microcanonical ensemble for string theory on AdS$_m\times$S^n, there is a phase transition between a black hole solution extended over the S^n and a solution localized on the S^n. If we think of this AdS$_m\times$S^n geometry as arising from the near-horizon limit of a black $m-2$ brane, the existence of this phase transition is puzzling. We present a resolution of this puzzle, and discuss its significance from the point of view of the dual $m-1$ dimensional field theory. We also discuss multi-black hole solutions in AdS.

Keywords: p-branes, Black Holes in String Theory.

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Contents

1. Introduction

The recently discovered AdS/CFT duality [2, 3, 4] between string theory in the bulk of anti-de Sitter spaces (times spheres) and large-N conformal field theories gives new insights into both the gauge theory and the nature of the bulk theory. In an early application, Witten [5] used this duality to relate the thermodynamics of asymptotically anti-de Sitter spaces [6] to the expected thermodynamics of the gauge theory. Recently, Banks, Douglas, Horowitz and Martinec [1] studied the microcanonical ensemble to determine the spectrum of string theory on these backgrounds in more detail. At high energies, the typical state is a Schwarzschild-AdS black hole. They argued that at lower energies, where the horizon radius is smaller than the cosmological scale, $r_+ < b$, the black hole will localize on the sphere due to the Gregory-Laflamme instability [7]. The typical state at lower energies will then be a D dimensional Schwarzschild black hole.

Certain Dp-branes have $AdS_{p+2}\times S^{D-p-2}$ spacetimes as their near-horizon geometries: the D3-brane, D1+D5 system, M2, and M5-branes have $(p, D) = (5, 10)$, $(1, 6)$, $(2, 11)$, $(5, 11)$ respectively. The existence of the above localization instability in the near-horizon region should then imply some instability of these Dp-branes. However, as the S^{D-p-2} corresponds to a sphere surrounding the Dp-brane, this is *not* the usual localization instability in the direction along the brane. Rather, it would imply that the stable solution is one in which the geometry is not spherically symmetric. This conclusion runs counter to the black hole no-hair theorems. We would also expect that any such asphericity would be radiated away. Thus there is apparently a puzzling contradiction between the expectations from the near-horizon region and the full asymptotically flat solution.

To understand the resolution of this puzzle, we consider the thermodynamics in more detail, especially the question of how it is affected by the asymptotic boundary conditions. In [5], Witten considered two sets of asymptotic boundary conditions. The conformal boundary was taken to be either $S^p \times S^1$ or $\mathbf{R}^p \times S^1$.[1] The discussion in [1] corresponds to the microcanonical version of the former choice, whereas the near-horizon limit of a Dp-brane corresponds to the latter [5, 8]. If we compactify the directions along the Dp-brane, the conformal boundary is $T^p \times S^1$.

In section 2, we consider the conformal branes, in particular the D3-brane, for which the near-horizon limit is $\mathrm{AdS}_5 \times \mathrm{S}^5$. We show that for a conformal boundary where the spatial part is T^3 (or $\mathbf{R}^3$), there is no localization instability on the S^5 in the microcanonical ensemble. The discussion is entirely similar for the M2- and M5-branes, and we state the results for these cases as well. We discuss the D1+D5-brane system, i.e., AdS_3, separately in section 3, as in this case, the distinction between different boundary conditions is more subtle. The spatial boundary is just a circle, but the six dimensional Schwarzschild black hole could be embedded in either an AdS_3 background, or in the $M = 0$ BTZ black hole. We give a physical argument for preferring the latter. In section 4 we discuss multi-black hole solutions in AdS backgrounds, and argue that the toroidal black holes cannot split up into multi-black holes. We conclude with a brief discussion.

2. Localization on S^{D-p-2} versus boundary conditions

In this section, we consider the asymptotically $\mathrm{AdS}_{p+2} \times \mathrm{S}^{D-p-2}$ spacetimes, which are related to D3-, D1+D5-, M2- and M5-branes for $(p, D) = (3, 10), (1, 6), (2, 11), (5, 11)$ respectively. We will begin by reviewing the case of a spacetime with spherical boundary conditions, which was discussed in [1] for $p = 3, 1$. For high energies, the dominant contribution comes from the Schwarzschild-AdS black hole (times an S^{D-p-2}) [5]. The metric of the asymptotically AdS factor is[2]

$$ds^2 = \left(\frac{r^2}{b^2} + 1 - \frac{w_{p+2}M}{r^{p-1}}\right) d\tau^2 + \left(\frac{r^2}{b^2} + 1 - \frac{w_{p+2}M}{r^{p-1}}\right)^{-1} dr^2 + r^2 d\Omega_p \,, \qquad (2.1)$$

where $w_{p+2} = 16\pi G_{p+2}/[p\mathrm{Vol}(S^p)] \sim G_D/b^{D-p-2}$, and G_d is the d-dimensional Newton constant. The radius b of the AdS factor depends on the brane:

$$\begin{aligned} b_{D3} &\sim (g_s N)^{1/4} \ell_s \,, & b_{M2} &\sim \ell_{11} N^{1/6} \,, \\ b_{M5} &\sim \ell_{11} N^{1/3} \,, & b_{D1+D5} &\sim (g_6 N_1 N_5)^{1/4} \ell_s \,, \end{aligned} \qquad (2.2)$$

[1]For pure AdS_{p+2}, the boundary is S^{p+1}, which is conformally equivalent to $\mathbf{R}^{p+1}$. $S^p \times S^1$ and $\mathbf{R}^p \times S^1$ are, however, not conformally equivalent; for instance, the former has a conformally invariant parameter, the ratio of the two radii, while the latter does not.

[2]We will write metrics in Euclidean signature, and use the canonical ensemble as a "trick" to calculate the entropy, but our physical interest is in the microcanonical ensemble.

JHEP12(1998)020

where ℓ_s is the string length and $\ell_{11} = \ell_s g_s^{1/3}$ is the eleven-dimensional Planck length. The event horizon of this asymptotically AdS black hole is at $r = r_+$, where r_+ solves the equation

$$\frac{r_+^2}{b^2} + 1 - \frac{w_{p+2} M}{r_+^{p-1}} = 0\,. \tag{2.3}$$

As shown in [6], the entropy is given by an expression familiar from asymptotically flat spaces,

$$S = \frac{1}{4G_{p+2}} r_+^p \mathrm{Vol}(S^p)\,. \tag{2.4}$$

There is no elementary expression for the entropy as a function of mass. If we rewrite the horizon position relation (2.3) as an expression for the mass,

$$M = \frac{p\mathrm{Vol}(S^p)}{16\pi G_{p+2}} \left(\frac{r_+^{p+1}}{b^2} + r_+^{p-1} \right), \tag{2.5}$$

we see that there are two limits of the parameter r_+/b in which there is a simple approximate expression for the entropy. Black holes whose horizon is large by comparison to the radius of curvature of the AdS_{p+2} have $r_+/b \gg 1$. In this case, the first term in the mass dominates, and hence $S \sim (b^{D-2}/G_D)\,(G_D M/b^{D-3})^{p/(p+1)}$. For small black holes, $r_+/b \ll 1$, the second term dominates, and so

$$S \sim \frac{1}{G_{p+2}} \left(G_{p+2} M\right)^{p/(p-1)} = \frac{b^p}{G_{p+2}} \left(\frac{G_{p+2} M}{b^{p-1}} \right)^{p/(p-1)} \sim \frac{b^{D-2}}{G_D} \left(\frac{G_D M}{b^{D-3}} \right)^{p/(p-1)}. \tag{2.6}$$

In the canonical ensemble, these small black holes have negative specific heat, and are unstable. However, we work in the microcanonical ensemble, where the energy (rather than the temperature of the heat bath) is fixed and this instability is absent. Instead, a different kind of instability is present.

On scales much less than the radius of curvature of the AdS_{p+2}, the spacetime away from the black hole horizon looks approximately like flat D-dimensional spacetime. The entropy S' of D-dimensional Schwarzschild black holes has a different dependence on mass than that for the small Schwarzschild-AdS_{p+2} black holes:

$$S' \sim \frac{1}{G_D} \left(G_D M\right)^{(D-2)/(D-3)} = \frac{b^{D-2}}{G_D} \left(\frac{G_D M}{b^{D-3}} \right)^{(D-2)/(D-3)}. \tag{2.7}$$

By comparing this with (2.6), we see that the small Schwarzschild-AdS black holes are entropically unstable, and will undergo a localization transition leading to a D-dimensional Schwarzschild black hole in this approximately flat region. The crossover happens when $r_+/b \sim 1$, and the entropy of the Schwarzschild black hole is larger for $r_+/b < 1$.

These properties of Schwarzschild-AdS black holes are of course all dependent on the form of the metric, which is in turn crucially dependent on the boundary conditions. We now turn to near-horizon limits of conformal branes in order to see if the puzzle persists in these geometries. The dominant contribution to the physics at high energies will come from a black hole with a toroidal horizon [9, 8, 5]. This black hole metric is obtained directly as the near-horizon limit of the brane metric. For the purposes of illustration we will specialize to the D3-brane case, and comment on the M-branes at the end of the section. The toroidal black hole metric is then

$$ds^2 = \frac{\ell_s^4 U^2}{b^2}\left[\left(1-\frac{U_0^4}{U^4}\right)d\tau^2 + dy^i dy_i\right] + \frac{b^2}{U^2}\left(1-\frac{U_0^4}{U^4}\right)^{-1} dU^2\,. \tag{2.8}$$

We take the y_i to be periodic, $y_i \equiv y_i + L$. We can of course change the periodicity in y_i and the value of U_0 by a coordinate transformation, but the combination

$$\rho_{0\,(D3)} \equiv \frac{\ell_s^2 U_0 L}{b} \tag{2.9}$$

is coordinate-invariant. This is the proper length of the compactified directions at the event horizon.

To calculate the entropy, we use the canonical ensemble. The conformal boundary is $T^3 \times S^1$, rather than $S^3 \times S^1$, and ρ_0 determines the ratio of the size of the T^3 to the size β of the S^1, which is the only conformally invariant boundary datum. In particular, in direct analogy with the analysis [5] of the S^3 case, we calculate the entropy by varying with respect to the invariant quantity

$$\gamma \equiv \frac{b\,\beta}{L} = \frac{\pi b^2}{\rho_0} \tag{2.10}$$

rather than the temperature β. The evaluation of the action follows the same lines as in [6]. We find

$$I = -\,\frac{1}{16 G_5}\rho_0^3\,. \tag{2.11}$$

As in [5], we use this action to approximate the partition function, and obtain the mass and then the entropy by varying with respect to γ. The mass of this toroidal black hole is then

$$M = \frac{1}{b}\frac{3b^3}{16\pi G_5}\left(\frac{\rho_0}{b}\right)^4 \sim \frac{b^7}{G_{10}}\left(\frac{\rho_0}{b}\right)^4. \tag{2.12}$$

Note that this differs from the excess energy over extremality by a dimensionless factor (L/b). The entropy is then calculated via

$$S = \gamma M - I = \left(\gamma\frac{\partial}{\partial\gamma} - 1\right) I\,, \tag{2.13}$$

which yields

$$S = \frac{b^3}{4G_5}\left(\frac{\rho_0}{b}\right)^3 \sim \frac{b^8}{G_{10}}\left(\frac{G_{10}M}{b^7}\right)^{3/4}. \tag{2.14}$$

From this we see the essential difference between the toroidal and spherical conformal boundaries: here the entropy as a function of mass is the same power law, $S \sim M^{3/4}$, for all horizon sizes. This difference results directly from the different form of the metric (2.8) as compared to (2.1). With our definition of mass, it is also consistent with the observation in [5] that this black hole (without the periodic identifications, i.e., $L \to \infty$) can be obtained from Schwarzschild-AdS by taking the large-mass limit of the latter. It also implies that the specific heat of these toroidal black holes is positive.

To see if there is a localization instability in the microcanonical ensemble of the type found for the spherical boundary conditions, we compare the toroidal solution to the $D = 10$ Schwarzschild black hole. Substituting $D = 10$ into the expression (2.7) for the Schwarzschild entropy, we find $S' \sim (b^8/G_{10})(G_{10}M/b^7)^{8/7}$. This is comparable to the toroidal black hole entropy (2.14) when $(G_{10}M/b^7) \sim 1$, and using the mass formula (2.12) we see that this happens when $\rho_0 \sim b$, i.e., when the horizon size is of order the cosmological scale. But while the entropy of the $D = 10$ Schwarzschild black hole varies more slowly with energy (mass) than Schwarzschild-AdS, it varies more rapidly than the entropy of the toroidal black hole. Therefore, even though the entropies agree when the horizon size is of order the cosmological scale, the $D = 10$ Schwarzschild entropy is lower at smaller energies and so there is *no* localization instability. (This also applies for $\mathbf{R}^3$, i.e., the $L \to \infty$ limit.) Of course, the fact that the $D = 10$ Schwarzschild entropy is larger for larger black holes does not make the large toroidal black holes entropically unstable either, because the transition to a $D = 10$ Schwarzschild black hole was possible only for black holes smaller than b, i.e., where we could not "see" the cosmological constant.

Although we have explicitly analyzed only the D3-brane, we can easily extend this to the M2- and M5-branes. In the case of the M2-brane, the nonextremality is parameterized by the function $(1 - U_0^3/U^3)$, because the variable U in which the asymptotic $\mathrm{AdS}_4 \times \mathrm{S}^7$ structure is manifest is related to the radial variable r by $r = U^{1/2}\ell_{11}^{3/2}$, rather than the more familiar D-brane relation $U = r/\ell_s^2$. In this case, the proper size of the horizon is given by

$$\rho_{0\,(M2)} \sim \frac{\ell_{11}^3 U_0 L}{b^2}\,. \tag{2.15}$$

For the M5-brane, nonextremality is parameterized by the function $(1 - U_0^6/U^6)$, because $r = U^2\ell_{11}^3$, and

$$\rho_{0\,(M5)} \sim \frac{\ell_{11}^{3/2} U_0 L}{b^{1/2}}\,. \tag{2.16}$$

For both M-branes, the inverse temperature scales as $\beta \sim \sqrt{N}/U_0$, and the conformally invariant boundary datum scales as $\gamma = \beta(b/L) \sim b^2/\rho_0$, as was the case for the D3-brane in (2.10). Then using the equations (2.2) for the radius b of the AdS, we find that the mass scales as $M \sim (1/b)(b^p/G_{p+2})(\rho_0/b)^{p+1} \sim (b^8/G_{11})(\rho_0/b)^{p+1}$, and the entropy as $S \sim (b^p/G_{p+2})(\rho_0/b)^p \sim (b^9/G_{11})(G_{11}M/b^8)^{p/(p+1)}$. Again, by comparing with the Schwarzschild entropy (2.7), which for $D = 11$ scales as $S' \sim (b^9/G_{11})(G_{11}M/b^8)^{9/8}$, we see that there is *no* S^{D-p-2} localization instability with a toroidal boundary, because the entropy of these small toroidal black holes dominates that of the eleven dimensional Schwarzschild black holes.

3. AdS$_3$ and two notions of mass

For the AdS$_3\times$S^3 spacetimes, which arise in the near-horizon limit of the D1+D5-brane system, the spherical and toroidal boundary conditions degenerate to a single case, where the spatial boundary is just a circle. The dominant contribution at high energies comes from the BTZ black hole [10],

$$ds^2 = (r^2/b^2 - G_3M)d\tau^2 + \frac{dr^2}{(r^2/b^2 - G_3M)} + r^2d\phi^2\,, \tag{3.1}$$

where M is the ADM mass, and the entropy is

$$S = \frac{\pi r_+}{2G_3} = \frac{\pi b\sqrt{G_3M}}{2G_3} \sim \frac{b^{5/2}\sqrt{G_6M}}{G_6}\,. \tag{3.2}$$

Pure AdS$_3$ is given by the BTZ black hole (3.1) with $G_3M = -1$. In the Euclidean approach to the calculation of this entropy, the action is calculated using the $M = 0$ black hole as a background.

We want to compare this to the entropy for a $D = 6$ Schwarzschild black hole, which is

$$S' \sim \frac{1}{G_6}(M'G_6)^{4/3}\,. \tag{3.3}$$

If we consider this black hole inserted into a pure AdS$_3$ ($\times$S^3) background, then we should take $M' = M + 1/G_3$, so that the ADM mass conjugate to time t is M. Alternatively, if the background geometry should be the $M = 0$ BTZ black hole ($\times$S^3), then $M' = M$. In [1], the former alternative was implicitly taken. There is then a localization transition at $M \sim 1/G_3$ between the BTZ black hole and a $D = 6$ Schwarzschild black hole embedded in AdS$_3$.

The BTZ black hole (3.1) is the near-horizon limit of a compactified black string; the compactified direction along the string becomes the angular direction in the BTZ solution. Pure AdS$_3$ cannot be obtained as the near-horizon limit of some regular string solution[3]; the mass parameter M in (3.1) is proportional to the energy above extremality of the string, which cannot be negative. We therefore argue that if we are considering this $M > 0$ BTZ spacetime as the near-horizon limit of a D1+D5-brane system, then we should compare to a $D = 6$ Schwarzschild black

[3]In this solution, the direction along the string must also be compact, if the compactified black string is to decay into it. AdS$_3$ is of course the near-horizon limit if this direction is not compactified.

JHEP12(1998)020

hole in an $M = 0$ BTZ background, not the AdS_3 background. In this case, the entropies are still comparable when $M \sim 1/G_3$, but as we lower M, the entropy of the $D = 6$ Schwarzschild black hole decreases more quickly than that of the BTZ black hole. Therefore, in the near-horizon limit of the D1+D5-brane system, there is *no* localization instability on S^3.

4. Multi-black hole instability

The black hole solutions (2.8,3.1) which appear when the boundary at infinity has topology $T^p \times S^1$ have the unusual property that the entropy grows less than linearly in the mass. It might appear that it would therefore be entropically favorable for these solutions to fragment into a number of smaller black holes of the same type. This would constitute a new instability for these solutions. This instability is also cause for concern, as we might be able to violate the Bekenstein bound if we had enough small black holes in a finite region.

For the BTZ black hole, there is an elegant proof that such an instability is in fact impossible: any pair of black holes in an asymptotically AdS_3 spacetime is always contained within a larger black hole [11, 12]. In any attempt to construct initial data describing a pair of black holes, if the separation between them is small, the spatial section is closed and there is no asymptotic region, while if the separation is larger, there is extra energy from separating the black holes and the radius of the resulting black hole is greater than the separation. This answer accords well with our intuition about the Bekenstein bound; whenever we try to violate the bound by packing a lot into a small volume, we find that the energy is so large that the whole system already lies inside a larger black hole.

In general, for any asymptotically AdS_{p+2} solution, we do not expect to be able to separate black holes which are large compared to the cosmological scale. What this means is that if we initially have a pair of large separated black holes, we expect they will merge after a time of order b, which is short compared with the characteristic evolution time $\sim \rho_0$ associated with the black holes. We therefore cannot treat the black holes as separate thermodynamic systems. In particular, we cannot apply the formula (2.14) for the entropy of a static black hole in this case. Note that this also explains why we should not be worried about such an instability for large black holes in the case with spherical boundary conditions, even though they also have an entropy which grows less than linearly in the mass.

In the case of the higher-dimensional toroidal black holes, we still have to worry about black holes with horizons smaller than the cosmological scale. As $\rho_0 < b$, the characteristic evolution timescale of the black holes is short, and the entropy should be well-approximated by adding the entropies (2.14) for the individual black holes.

We should try to construct initial data corresponding to such multi-black hole solutions. To simplify the problem, we assume that the solution remains independent of all but one of the y_i; that is, we just separate the black holes in say the U, y_1 plane. The general form of the initial data is then

$$ds^2 = f(U, y_1)dU^2 + g(U, y_1)U^2 dy_1^2 + h(U, y_1)dy_i^2 \,. \tag{4.1}$$

Since we assume the solutions are independent of $p-1$ of the y_i, we can eliminate these dimensions by Kaluza-Klein reduction to obtain an equivalent three-dimensional problem. The $p+2$ dimensional black holes have non-constant curvature, so this three-dimensional problem is not equivalent to the BTZ case. From the three-dimensional point of view, this is because these black hole solutions involve a non-trivial value for the scalar field arising from the dy_i^2 part of the metric.

If we consider the initial data for the black hole (2.8) coming from the near-horizon limit of the D3-brane, then the U, y_1 part of the metric is the initial data for the three-dimensional metric. This two-dimensional surface has curvature

$$R = -\frac{2}{b^2}\left(1 + \frac{U_0^4}{U^4}\right). \tag{4.2}$$

In the special case $U_0 = 0$, the curvature is constant; the three-dimensional solution obtained by reduction of (2.8) is then the $M = 0$ BTZ black hole. We might have expected to get AdS_3 instead, as $U_0 = 0$ is supposed to be pure anti-de Sitter space. However, because we are taking toroidal boundary conditions at infinity, the $U_0 = 0$ solution is actually AdS space with a discrete set of identifications. In general, the non-constant part of the curvature is important only near the horizon of the black hole. If we consider a black hole which is much smaller than the cosmological scale, the proper size of the y_1 direction becomes small compared to the cosmological scale, signaling the presence of a black hole, long before we get to the region where the curvature due to the black hole becomes important. Thus, outside of a small region near the horizon, the initial data surface looks like the initial data for the $M = 0$ BTZ black hole.

For initial data describing more than one small black hole, there is a region away from the horizon, but still on scales small compared to the cosmological scale, where we can argue that the metric looks like the $M = 0$ BTZ black hole. Thus, the full initial data surface (apart from the small region around each horizon) should be well-approximated by the multi-BTZ case. But we know from [11, 12] that in that case, there is a larger black hole horizon encompassing the others. Thus, it seems reasonable to conjecture that for black holes small compared to the cosmological scale in the higher-dimensional case, the same is true. That is, we conjecture that there is no such instability for these black holes either.

There is no contradiction between this conjecture and the fact that we can separate branes. The near-horizon geometry of two groups of conformal branes with a small separation between them is not a direct product of the form $AdS_m \times S^n$, so it is not included in this discussion, where we have been considering just the AdS part.

5. Discussion

Our main point is that the S^{D-p-2} localization phase transition observed in [1], which was seen when the horizon size gets down to the cosmological scale, does not occur for the spacetimes which arise in the near-horizon limit of Dp- or M-branes. This transition arises for S^p spatial boundary conditions, but the near-horizon limit gives T^p boundary conditions (or the large-radius limit $\mathbf{R}^p$). Thus, this localization transition does *not* imply an instability of the Dp- or M-brane solutions, in agreement with the general expectation that there is no such instability.

We also considered a potential instability for AdS black holes to break up into smaller black holes, and argued that it does not occur either. In any initial data which describes several small black holes in an asymptotically AdS spacetime, there should be a larger black hole horizon which encompasses them. This is consistent with the fact that a group of Dp-branes can break up into smaller ones, because the near-horizon geometry does not retain the simple direct product form when the Dp-branes break up.

For the case with S^p spatial boundary, the black hole correspondence principle tells us that there are further distinct phases as the mass of the system gets even lower. As discussed in [1], when the horizon size of the D dimensional Schwarzschild black hole gets down to the string scale, the system goes into a Hagedorn, or long string, phase. At still lower energies, we see a gas of supergravitons.

For the $T^p \times S^1$ case, there are further phases at energies below that of the toroidal black hole, but the structure is rather different, and neither the long string or AdS supergraviton gas phases will appear. For the T^3 case, the additional phases were analyzed in [13]. The key is that there is a torus with a U-dependent size. At low temperature, the horizon scale ρ_0 is smaller than the string scale, so we must T-dualize at some $U > U_0$, resulting in a "smeared" D0-brane spacetime. Note that this transition is at a gauge theory temperature which is higher than that at which D3-brane finite-size effects kick in, taking into account D3-brane fractionation [14]. At low enough temperature, there is a localization to a D0-brane spacetime at smaller U. Note that in this phase, the $D = 10$ black hole carries Ramond-Ramond charge, unlike the $D = 10$ Schwarzschild black hole which arises for the S^3 boundary conditions. At even lower temperatures, there are further phases with more eleven-dimensional structure. For the other T^p cases, there should similarly be additional phases which appear when ρ_0 is less than the relevant length scale.

From the gauge theory point of view, the localized black hole phase is particularly interesting. This phase appears only for gauge theory on the p-sphere at strong coupling. It breaks the $SO(6)$ R-symmetry, but without breaking spherical symmetry on the S^p. This is similar to the Coulomb phase which appears for toroidal boundary conditions, but the two are otherwise very different. The most interesting aspect of the localized black hole phase is that the Schwarzschild black hole has a negative specific heat. This is the only instance of which we are aware in which a black hole with negative specific heat is represented in the gauge theory. It would be interesting to investigate this further.

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JHEP12(1998)020